

Exercise 20

For the following exercises, use synthetic division to find the quotient.

$$(-6x^3 + x^2 - 4) \div (2x - 3)$$

Solution

Synthetic division only works if the divisor is of the form $x - k$. Multiply the numerator and denominator by $1/2$ to make it so.

$$\frac{-6x^3 + x^2 - 4}{2x - 3} = \frac{-6x^3 + x^2 - 4}{2x - 3} \times \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{-3x^3 + \frac{1}{2}x^2 - 2}{x - \frac{3}{2}}$$

Solving $x - \frac{3}{2} = 0$ gives $x = \frac{3}{2}$; this is the number that goes on the top left. Write out all the coefficients of the dividend to the right.

$$\begin{array}{r|rrrr} \frac{3}{2} & -3 & \frac{1}{2} & 0 & -2 \\ \hline & & & & \end{array}$$

Bring down the leading coefficient.

$$\begin{array}{r|rrrr} \frac{3}{2} & -3 & \frac{1}{2} & 0 & -2 \\ \hline & -3 & & & \end{array}$$

Multiply the top left number by the number brought down and put the result under the second coefficient of the dividend.

$$\begin{array}{r|rrrr} \frac{3}{2} & -3 & \frac{1}{2} & 0 & -2 \\ \hline & -3 & \frac{9}{2} & & \end{array}$$

Add the numbers in the second column.

$$\begin{array}{r|rrrr} \frac{3}{2} & -3 & \frac{1}{2} & 0 & -2 \\ \hline & & 9 & & \\ & & -\frac{9}{2} & & \\ \hline & -3 & -4 & & \end{array}$$

Multiply this sum of the second column by the top left number and put it in the next column.

$$\begin{array}{r|rrrr} \frac{3}{2} & -3 & \frac{1}{2} & 0 & -2 \\ \hline & & 9 & & \\ & & -\frac{9}{2} & -6 & \\ \hline & -3 & -4 & & \end{array}$$

Add the numbers in the third column.

$$\begin{array}{r|rrrr} \frac{3}{2} & -3 & \frac{1}{2} & 0 & -2 \\ \hline & & 9 & & \\ & & -\frac{9}{2} & -6 & \\ \hline & -3 & -4 & -6 & \end{array}$$

Multiply this sum of the third column by the top left number and put it in the next column.

$$\begin{array}{r|rrrr} \frac{3}{2} & -3 & \frac{1}{2} & 0 & -2 \\ \hline & & \frac{9}{2} & -6 & -9 \\ \hline & -3 & -4 & -6 & \end{array}$$

Add the numbers in the fourth column.

$$\begin{array}{r|rrrr|r} \frac{3}{2} & -3 & \frac{1}{2} & 0 & -2 & \\ \hline & & \frac{9}{2} & -6 & -9 & \\ \hline & -3 & -4 & -6 & -11 & \end{array}$$

This final result is the remainder, and the numbers to the left are the coefficients of the quotient, which is $-3x^2 - 4x - 6$.

$$(-6x^3 + x^2 - 4) \div (2x - 3) = -3x^2 - 4x - 6 + \frac{-11}{x - \frac{3}{2}}$$